\textbf{thetas} = \{1.628814629, 1.598374576, 1.568798654, 1.547181935, 1.525301696, 1.478323485, 1.434171566, 1.379201972, 1.317838286, 1.26194459, 1.198314814, 1.135126452, 1.074120306, 1.018986493, 0.969829573, 0.905741116, 0.851885256, 0.779456873, 0.67633202, 0.600506019, 0.52832569, 0.46302288, 0.390050894\}

\textbf{values} = \{n \rightarrow 2.3104, a \rightarrow 32.7557\}

\{n \rightarrow 2.3104, a \rightarrow 32.7557\}

\textbf{s} = a \theta^{1 / n} \text{Hypergeometric2F1}\left[{-1 / 2, 1 / (2 n), 1 + 1 / (2 n), -n^2 \theta^2}\right]

\text{a \theta^{1 / n} \text{Hypergeometric2F1}\left[-1 / 2, 1 / (2 n), 1 + 1 / (2 n), -n^2 \theta^2\right]}
s /. values /. theta \[\rightarrow\] thetas \[//\] MatrixForm

\[s0 = \%[[1]]\]

\[
\begin{bmatrix}
67.9446 \\
66.6748 \\
65.4494 \\
64.559 \\
63.6623 \\
61.7527 \\
59.9774 \\
57.7937 \\
55.391 \\
53.235 \\
50.8182 \\
48.4583 \\
46.2176 \\
44.2244 \\
42.4726 \\
40.2238 \\
38.3643 \\
35.9051 \\
32.4792 \\
30.0077 \\
27.6809 \\
25.5844 \\
23.2277
\end{bmatrix}
\]

\[67.9446\]

\[r = a (1 + n^2 \theta^2)^{(3/2)} / (n \theta^{(1 - 1/n)} (1 + n + n^2 \theta^2))\]

\[
a \theta^{-1/n} (1 + n^2 \theta^2)^{3/2} \\
---------------------------------
\frac{n (1 + n + n^2 \theta^2)}{}
\]
\textbf{Mathematica Code}

```
In[1]:= r /. values /. theta \[RightTendsto] thetas // MatrixForm

\begin{verbatim}
( 36.3253
  35.9055
  35.4916
  35.1852
  34.8717
  34.1867
  33.5278
  32.6859
  31.7168
  30.806
  29.7352
  28.6351
  27.5375
  26.5155
  25.5808
  24.3305
  23.2546
  21.7782
  19.6455
  18.0902
  16.664
  15.4665
  14.3094)
\end{verbatim}

Out[2]=

\textbf{Plot}

```

\textbf{Plot}[s /. values /. theta \[RightTendsto] x, \{x, 0, 6.5\}]
```
Plot[r/.values/.theta→x,{x,0,6.5}]

logvalues = {a -> 2.3470, b -> .4539}
{a → 2.347, b → 0.4539}

slog = a Sqrt[1 + b^2] Exp[b t] / b

\[
a \frac{\sqrt{1 + b^2}}{b} e^{bt}
\]

rlog = a Sqrt[1 + b^2] Exp[b t]

\[
a \sqrt{1 + b^2} e^{bt}
\]
\text{tvalues} = \{5.949368226, \\
5.916305259, \\
5.883500689, \\
5.859614478, \\
5.834924816, \\
5.781015333, \\
5.729627125, \\
5.664809383, \\
5.590574456, \\
5.521592788, \\
5.440581862, \\
5.359524152, \\
5.278537577, \\
5.203598777, \\
5.134747049, \\
5.043009242, \\
4.963496788, \\
4.853631719, \\
4.69103705, \\
4.564607637, \\
4.436922348, \\
4.315018521, \\
4.168544148\}

\{5.94937, 5.91631, 5.8835, 5.85961, 5.83492, 5.78102, 5.72963, \\
5.66481, 5.59057, 5.52159, 5.44058, 5.35952, 5.27854, 5.2036, 5.13475, \\
5.04301, 4.9635, 4.85363, 4.69104, 4.56461, 4.43692, 4.31502, 4.16854\}
\[\text{slog} \rightarrow \text{logvalues} \rightarrow \text{tvalues} \rightarrow \text{MatrixForm}\]

\[\text{s0log} = \%[[1]]\]

\[
\begin{bmatrix}
84.5294 \\
83.2703 \\
82.0396 \\
81.155 \\
80.2506 \\
78.3107 \\
76.5052 \\
74.2872 \\
71.8257 \\
69.6117 \\
67.0985 \\
64.6746 \\
62.3404 \\
60.2556 \\
58.4016 \\
56.0197 \\
54.0339 \\
51.4055 \\
47.7483 \\
45.0853 \\
42.5466 \\
40.2564 \\
37.667
\end{bmatrix}
\]

84.5294

\[\text{xlog} \rightarrow \text{logvalues} \rightarrow \text{tvalues} \rightarrow \text{MatrixForm}\]

\[
\begin{bmatrix}
38.3679 \\
37.7964 \\
37.2378 \\
36.8362 \\
36.4257 \\
35.5452 \\
34.7257 \\
33.7189 \\
32.6017 \\
31.5967 \\
30.456 \\
29.3558 \\
28.2963 \\
27.35 \\
26.5085 \\
25.4273 \\
24.526 \\
23.3329 \\
21.6729 \\
20.4642 \\
19.3119 \\
18.2724 \\
17.097
\end{bmatrix}
\]
\textbf{ArchemedianArcLengthCalculation.nb}

\begin{verbatim}
ParametricPlot[
\{s0 - s /. \text{values} /. \theta \rightarrow x / 2, r /. \text{values} /. \theta \rightarrow x / 2\},
\{s0 log - slog /. \logvalues /. t \rightarrow x, r log /. \logvalues /. t \rightarrow x\}, \{x, .1, 6.28\}]
\end{verbatim}

\begin{verbatim}
ParametricPlot[
\{-s0 log + slog /. \logvalues /. t \rightarrow x, r log /. \logvalues /. t \rightarrow x\}, \{x, .1, 6.28\}]
\end{verbatim}
thvalues = {0.5000, 0.5186, 0.5367, 0.5498, 0.5630, 0.5912, 0.6174, 0.6496, 0.6851, 0.7169, 0.7526, 0.7875, 0.8207, 0.8501, 0.8760, 0.9092, 0.9366, 0.9729, 1.0232, 1.0595, 1.0937, 1.1246, 1.1595}

{0.5, 0.5186, 0.5367, 0.5498, 0.563, 0.5912, 0.6174, 0.6496, 0.6851, 0.7169, 0.7526, 0.7875, 0.8207, 0.8501, 0.876, 0.9092, 0.9366, 0.9729, 1.0232, 1.0595, 1.0937, 1.1246, 1.1595}

rCloth = aCloth / Pi / th

sCloth = th * aCloth

aCloth th
rCloth /. aCloth \[\rightarrow\] 75.678 /. th \[\rightarrow\] thvalues // MatrixForm

\[
\begin{pmatrix}
48.1781 \\
46.4502 \\
44.8837 \\
43.8142 \\
42.787 \\
40.746 \\
39.0169 \\
37.0829 \\
35.1614 \\
33.6017 \\
32.0078 \\
30.5893 \\
29.3518 \\
28.3367 \\
27.4989 \\
26.4948 \\
25.7197 \\
24.7601 \\
23.5429 \\
22.7362 \\
22.0253 \\
21.4201 \\
20.7754
\end{pmatrix}
\]

sCloth /. aCloth \[\rightarrow\] 75.678 /. th \[\rightarrow\] thvalues // MatrixForm

s0Cloth = %[[1]]

\[
\begin{pmatrix}
37.839 \\
39.2466 \\
40.6164 \\
41.6078 \\
42.6067 \\
44.7408 \\
46.7236 \\
49.1604 \\
51.847 \\
54.2536 \\
56.9553 \\
59.5964 \\
62.1089 \\
64.3339 \\
66.2939 \\
68.8064 \\
70.88 \\
73.6271 \\
77.4337 \\
80.1808 \\
82.769 \\
85.1075 \\
87.7486
\end{pmatrix}
\]

37.839
Needs["PlotLegends`"]

ParametricPlot[
  {s0Cloth + sCloth /. aCloth -> 75.678 /. th -> x/6,
   rCloth /. aCloth -> 75.678 /. th -> x/6},
  {s0log - slog /. logvalues /. t -> x, rlog /. logvalues /. t -> x},
  {s0 - s /. values /. theta -> x/2, r /. values /. theta -> x/2},
  {x, .1, 8.28}, PlotRange -> {{-5, 50}, {10, 60}},
  PlotStyle -> {{Thick, Black}, {Thick, Dashed}, {Thick, Dotted}},
  Frame -> True,
  FrameLabel -> {"Distance (m)", "Radius of Curvature (m)"},
  Axes -> False,
  PlotLegend -> {"Clothoid Spiral", "Logarithmic Spiral", "Archimedean Spiral"},
  LegendShadow -> None]